Theory of Factorial Design: Single- and Multi-Stratum Experiments

Notes

- Cover art: This figure appears on p.281. It merges two Hasse diagrams representing the treatment and block structures in a blocked experiment for two 2-level treatment factors A and B. The circles correspond to the treatment main effects and interactions, and the bullets correspond to the three unit factors (universal, block, and equality factors) that make up the block structure of a block design. The merged Hasse diagram shows that the interaction of A and B is confounded with blocks. For details, see Example 13.13.

- P.49, line 17: Instead of "On the other hand, by the orthogonality of $\mathcal{F}_1$ and $\mathcal{F}_2$, (4.17) holds", it is better to say "Suppose $\mathcal{F}_1$ and $\mathcal{F}_2$ are orthogonal. Then (4.17) holds".

- P.78, the last two lines: The words "columns" here refer to the $s_1 \cdots s_n \times 1$ columns consisting of the coefficients in the contrasts defining main effects of the treatment factors, not the columns of the matrix in (6.20).

- P.114, (7.13): It is not explicitly stated here, but as in (7.9), the first $n - q$ columns of the design key matrix correspond to the subplot unit factors and the last q columns correspond to the whole-plot unit factors.

- P.131, the paragraph following Proposition 8.14: By the multiplication table, we mean the $s \times s$ matrix with the rows and columns corresponding to the elements of GF(s) such that the (i, j) entry is equal to the product of the element corresponding to the ith row and that corresponding to the jth column.

- P.131, line 2 from the bottom and the rest of Section 8.8: As mentioned, here $\oplus$ denotes the Kronecker sum (of matrices), not to be confused with the direct sum (of vector spaces).

- P.136, lines 20-22: It is more precise to say "Then we have an N-point Latin hypercube that is space-filling in low dimensional projections in the sense that in all h-dimensional projections, $h \leq t$, each cell of $s \times \cdots \times s$ grids contains the same number of design points."

- P.136, lines 2 and 3 from the bottom: It is more precise to say "each cell of $s \times s$ grids contains the same number of design points, but for one constructed from an SOA$(N, (s^3)^n, 3)$, this projection property holds for $s^2 \times s$ and $s \times s^2$ grids as well. "

- P.147, Theorem 9.3: We caution the readers that, for convenience, this theorem is stated for the case where the first $n - p$ factors are basic factors. However, the theorem merely establishes the existence of a set of basic factors. In general, the first $n - p$ factors may not form a set of basic factors.

- P.159 and P.160: Throughout these two pages, it is better to replace n with another letter since elsewhere in the book n is used to denote the number of treatment factors.

- P.161, lines 14 and 15 from the bottom: Instead of "furthermore let $W_i(C)$ be the number of codewords $\mathbf{x}$ with $w(\mathbf{x}) = i$, then $W_i(\mathbf{x}) = W_i(C)$ for all $\mathbf{x} \in C$", it is better to say "furthermore, $W_i(\mathbf{x})$ does not depend on $\mathbf{x}$, and so we have $W_i(\mathbf{x}) = W_i(C)$ for all $\mathbf{x} \in C$. In this case, $W_i(C)$ is also the number of codewords with Hamming weight equal to i".

- P.181, line 15 from the bottom: More precisely, $\overline{d}$ may consist of copies of a linear code.

- P.189, Lemma 10.11. This is a special case (the regular design version) of Lemma 15.20 on p.347. For the sum over i in Lemma 15.20, if $i > t$, then since $j \leq t$, we have $j - i < 0$; it follows that the expression inside the brackets is zero. Therefore the sum is actually over the range $0 \leq i \leq \min(n, t)$, as in Lemma 10.11. The same remark applies to (11.27) on p.220.

- P.191, lines 7-15: That if $\boldsymbol{\alpha}$ is stationary, then the covariance matrix of the factorial effects is diagonal and their variances are the eigenvalues of $\text{cov}(\boldsymbol{\alpha})$ multiplied by a constant can also be proved by using Theorem 12.7. See Exercise 12.5.

- P.197, lines 13 and 14 from the bottom: Instead of "none of the $2n+1$ effects $A_1, \ldots, A_{n+1}, A_1A_{n+1}, \ldots, A_nA_{n+1}$ is aliased", it is better to say "no two of the $2n + 1$ effects $A_1, \ldots, A_{n+1}, A_1A_{n+1}, \ldots, A_nA_{n+1}$ are aliased with each other".

- P.200, Theorem 11.7: The proof of this theorem is left as an exercise (Exercise 11.2).

- P.204, lines 6 and 12: Although there is only one maximal 2^{13-7} design, there are two maximal 2^{26-19} designs. Since $26 < 128/4 + 2$, Theorem

11.12 does not apply to the case $n = 26$ and $N = 128$. Not every maximal 2^{26-19} design of resolution IV is the double of a maximal 2^{13-7} design.

- P.207, Theorem 11.19: We remind readers that, as mentioned at the beginning of this section, unlike in (8.18), here the foldover plan does not include an extra factor.

- P.240, line 10: The boldfaced $\mathbf{n}_1\mathbf{n}_2$ is to emphasize that $1 + v_1 + n_1 v_2$ must be replaced by $n_1 n_2$, the algebraic sum of all its terms.

- P.245, line 17: That if $\mathcal{G} \preceq \mathcal{F}$ does not hold, then $W_{\mathcal{F}}$ is orthogonal to $V_{\mathcal{G}}$ can be seen as follows. By (12.29), $V_{\mathcal{G}} = \bigoplus_{\mathcal{F}' \in \mathfrak{B}: \mathcal{G} \preceq \mathcal{F}'} W_{\mathcal{F}'}$. Since the W spaces are mutually orthogonal, we have $V_{\mathcal{G}} \perp W_{\mathcal{H}}$ for all $\mathcal{H}$'s such that $\mathcal{G} \preceq \mathcal{H}$ does not hold. In particular, $V_{\mathcal{G}} \perp W_{\mathcal{F}}$.

- P.261, lines 9 and 10 from the bottom: This sentence is not correct and has been corrected in the Errata. The splitting effects include all the factorial effects defined by nonzero linear combinations of $\mathbf{a}_{n_1+1}, \ldots, \mathbf{a}_q$, and their generalized interactions with the factorial effects of whole-plot treatment factors. Here only the factorial effects defined by nonzero linear combinations of $\mathbf{a}_{n_1+1}, \ldots, \mathbf{a}_q$ are confounded with blocks.

- Section 13.6: In this section, we only consider designs constructed by the method presented in Section 13.5. This assumption applies to Theorems 13. 3 and 13.4, though it is not explicitly stated.

- P. 306, lines 3 and 5 from the bottom: It is more precise to say that the row (column) design would be reduced to a $replicated$ $s^{n_1-p_1-k}$ ($s^{n_2-p_2-l}$) fraction.

- P.326, line 3 from the bottom: Instead of "the unblocked 2^{n-p} fractional factorial design d^*", it is better to say "the unblocked version of the design d^*"; that is, the unblocked 2^{n-p} fractional factorial design constructed from the n points in T_{d^*}".

- P.340, Equation (15.9): Compared with (15.8), the expression of $B_k(d)$ in (15.9) has an extra term $s_1 \cdots s_n$, which is equal to 2^n for two-level designs. It is needed here so that, for two-level designs, the expression in (15.9) is equal to that in (15.8); this is because $\mathbf{p}^{\mathbf{z}}$ is normalized to have unit length, whereas the $B_k(d)$ in (15.8) is based on model matrices with non-normalized columns consisting of 1's and -1's.

- P.346, Theorem 15.19: The claimed minimum moment aberration (and generalized minimum aberration) is over the designs with all the treatment factor levels appearing $\lfloor N/s \rfloor$ or $\lfloor N/s \rfloor + 1$ times. The lower bound in (15.13) applies only to the designs with all the treatment factor levels appearing N/s times.

- P.350, line 13: This is the case if the added foldover runs form a regular fraction.