Genotypic Distributions of Three and Four loci under Self and Random Mating for the Derivation of Correlations between Pairwise QTL

Chen-Hung Kao and Miao-Hui Zeng

For any pair of QTL, they can be located in neighboring or non-neighboring marker intervals. For neighboring case, distribution of three flanking marker loci is needed to compute the correlation between these two QTL. For non-neighboring case, distributions of four markers is required to obtain their correlation. In general, there are 2^m different gametic genotypes and $2^{2m-1} + 2^m/2$ zygotic genotypes for m loci in the populations from two inbred lines. For example, there are 4, 8 and 16 gametic genotypes and 10, 36 and 136 zygotic genotypes for $m = 2$, 3 and 4. As different populations will undergo various number of meiosis cycle, the distributions of gametic and zygotic genotypes will vary. For selfing, Haldane and Waddington (1931) formulated the transition equations of the ten genotypic frequencies for $m = 2$. Kao and Zeng (2009) obtained the transition equations of the 36 genotypic frequencies for $m = 3$. For random mating, Geiringer's approach (1944) can be used to obtain the frequencies of the gametic genotypes for any m, and, then, to obtain the genotypic frequencies. The procedures of obtaining transition equations for $m = 4$ are given below. Let 1 and 0 represent the capital and small-letter alleles, respectively, from P_1 and P_2, so that the configurations of the 16 gametes can be represented as $\underline{1111}$, $\underline{0000}$, $\underline{1110}$, $\underline{0001}$, $\underline{1101}$, $\underline{0010}$, $\underline{1011}$, $\underline{0100}$, $\underline{0111}$, $\underline{1000}$, $\underline{1100}$, $\underline{0011}$, $\underline{1010}$, $\underline{0101}$, $\underline{1001}$, and $\underline{0110}$. In the F$_2$ population, these 16 gamete frequencies can be obtained under Haldane map function (using the Markov property), and they are $P(\underline{1111}) = P(\underline{0000}) = (1 - r_1)(1 - r_2)(1 - r_3)/2$, where r_1, r_2 and r_3 are the recombination rates between the first and second genes, between the second and third genes, and the third and four genes, respectively. The other frequencies are $P(\underline{1110}) = P(\underline{0001}) = (1 - r_1)(1 - r_2)r_3/2$, $P(\underline{1101}) = P(\underline{0010}) = (1 - r_1)r_2 r_3/2$, $P(\underline{1011}) = P(\underline{0100}) = r_1 r_2(1 - r_3)/2$, $P(\underline{0111}) = P(\underline{1000}) = r_1(1-r_2)(1-r_3)/2$, $P(\underline{1100}) = P(\underline{0011}) = (1-r_1)r_2(1-r_3)/2$, $P(\underline{1010}) = P(\underline{0101}) = r_1 r_2 r_3/2$, and $P(\underline{1001}) = P(\underline{0110}) = r_1(1 - r_2)r_3/2$, respectively. The unification of these 16 gametes frequencies

can produce the 136 different zygotes in a population. Because of symmetry, the complementary pairs have the same frequencies, there are 72 different genotypic frequencies. Let the 136 zygotic genotypes occur in the proportions in a population:

$$A_1 \quad \frac{1111}{1111} \; \frac{0000}{0000} \qquad A_2 \quad \frac{1111}{1110} \; \frac{0000}{0001} \qquad A_3 \quad \frac{1110}{1110} \; \frac{0001}{0001}$$

$$A_4 \quad \frac{1111}{1011} \; \frac{0000}{0100} \qquad A_5 \quad \frac{1111}{1010} \; \frac{0000}{0101} \qquad A_6 \quad \frac{1110}{1011} \; \frac{0001}{0100}$$

$$A_7 \quad \frac{1110}{1010} \; \frac{0001}{0101} \qquad A_8 \quad \frac{1011}{1011} \; \frac{0100}{0100} \qquad A_9 \quad \frac{1011}{1010} \; \frac{0100}{0101}$$

$$A_{10} \quad \frac{1010}{1010} \; \frac{0101}{0101} \qquad A_{11} \quad \frac{1111}{1101} \; \frac{0000}{0010} \qquad A_{12} \quad \frac{1111}{1100} \; \frac{0000}{0011}$$

$$A_{13} \quad \frac{1110}{1101} \; \frac{0001}{0010} \qquad A_{14} \quad \frac{1110}{1100} \; \frac{0001}{0011} \qquad A_{15} \quad \frac{1111}{1001} \; \frac{0000}{0110}$$

$$A_{16} \quad \frac{1101}{1011} \; \frac{0010}{0100} \qquad A_{17} \quad \frac{1111}{1000} \; \frac{0000}{0111} \qquad A_{18} \quad \frac{1100}{1011} \; \frac{0011}{0100}$$

$$A_{19} \quad \frac{1010}{1101} \; \frac{0101}{0010} \qquad A_{20} \quad \frac{1001}{1110} \; \frac{0110}{0001} \qquad A_{21} \quad \frac{1110}{1000} \; \frac{0001}{0111}$$

$$A_{22} \quad \frac{1100}{1010} \; \frac{0011}{0101} \qquad A_{23} \quad \frac{1011}{1001} \; \frac{0100}{0110} \qquad A_{24} \quad \frac{1011}{1000} \; \frac{0100}{0111}$$

$$A_{25} \quad \frac{1010}{1001} \; \frac{0101}{0110} \qquad A_{26} \quad \frac{1010}{1000} \; \frac{0101}{0111} \qquad A_{27} \quad \frac{1101}{1101} \; \frac{0010}{0010}$$

$$A_{28} \quad \frac{1101}{1100} \; \frac{0010}{0011} \qquad A_{29} \quad \frac{1100}{1100} \; \frac{0011}{0011} \qquad A_{30} \quad \frac{1101}{1001} \; \frac{0010}{0110}$$

$$A_{31} \quad \frac{1101}{1000} \; \frac{0010}{0111} \qquad A_{32} \quad \frac{1100}{1001} \; \frac{0011}{0110} \qquad A_{33} \quad \frac{1100}{1000} \; \frac{0011}{0111}$$

$$A_{34} \quad \frac{1001}{1001} \; \frac{0110}{0110} \qquad A_{35} \quad \frac{1001}{1000} \; \frac{0110}{0111} \qquad A_{36} \quad \frac{1000}{1000} \; \frac{0111}{0111}$$

$$A_{37} \quad \frac{1111}{0111} \; \frac{0000}{1000} \qquad A_{38} \quad \frac{1111}{0110} \; \frac{0000}{1001} \qquad A_{39} \quad \frac{1110}{0111} \; \frac{0001}{1000}$$

$$A_{40} \quad \frac{1110}{0110} \; \frac{0001}{1001} \qquad A_{41} \quad \frac{1111}{0011} \; \frac{0000}{1100} \qquad A_{42} \quad \frac{1011}{0111} \; \frac{0100}{1000}$$

A_{43} $\frac{1111}{0010}$ $\frac{0000}{1101}$ A_{44} $\frac{1011}{0110}$ $\frac{0100}{1001}$ A_{45} $\frac{1010}{0111}$ $\frac{0101}{1000}$

A_{46} $\frac{1110}{0011}$ $\frac{0001}{1100}$ A_{47} $\frac{1110}{0010}$ $\frac{0001}{1101}$ A_{48} $\frac{1010}{0110}$ $\frac{0101}{1001}$

A_{49} $\frac{1011}{0011}$ $\frac{0100}{1100}$ A_{50} $\frac{1011}{0010}$ $\frac{0100}{1101}$ A_{51} $\frac{1010}{0011}$ $\frac{0101}{1100}$

A_{52} $\frac{1010}{0010}$ $\frac{0101}{1101}$ A_{53} $\frac{1111}{0101}$ $\frac{0000}{1010}$ A_{54} $\frac{1101}{0111}$ $\frac{0010}{1000}$

A_{55} $\frac{1111}{0100}$ $\frac{0000}{1011}$ A_{56} $\frac{1100}{0111}$ $\frac{0011}{1000}$ A_{57} $\frac{1110}{0101}$ $\frac{0001}{1010}$

A_{58} $\frac{1101}{0110}$ $\frac{0010}{1001}$ A_{59} $\frac{1110}{0100}$ $\frac{0001}{1011}$ A_{60} $\frac{1100}{0110}$ $\frac{0011}{1001}$

A_{61} $\frac{1111}{0001}$ $\frac{0000}{1110}$ A_{62} $\frac{1011}{0101}$ $\frac{0100}{1010}$ A_{63} $\frac{1101}{0011}$ $\frac{0010}{1100}$

A_{64} $\frac{1001}{0111}$ $\frac{0110}{1000}$ A_{65} $\frac{1111}{0000}$ A_{66} $\frac{1000}{0111}$

A_{67} $\frac{1100}{0011}$ A_{68} $\frac{1010}{0101}$ A_{69} $\frac{1110}{0001}$

A_{70} $\frac{1011}{0100}$ A_{71} $\frac{1001}{0110}$ A_{72} $\frac{1101}{0010}$

Note that A_1 denote the frequency of $\frac{1111}{1111}$ or $\frac{0000}{0000}$ genotypes, i.e. $A_1 = P(\frac{1111}{1111})$ or $A_1 = P(\frac{0000}{0000})$. If random mating persists after F_2 to produce AI populations, the transition equations for the frequencies of the 16 gametic genotypes can be derived using Geiringer's approach (1944), and, in turn, to obtain the 136 genotypic frequencies. If selfing persists after F_2 to generate RI populations, the 72 transition equations for the frequencies of the 136 genotypes are shown below. For $\frac{1111}{1111}$ genotype, the equation is

$$
\begin{aligned}
A_1' =\ & A_1 + \frac{1}{4}A_2 + \frac{1}{4}A_4 + \frac{(1-f_2)^2}{4}A_5 + \frac{f_2^2}{4}A_6 + \frac{1}{4}A_{11} + \frac{(1-r_3)^2}{4}A_{12} + \frac{r_3^2}{4}A_{13} + \frac{(1-r_2)^2}{4}A_{15} \\
& + \frac{r_2^2}{4}A_{16} + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{17} + \frac{[r_2(1-r_3)]^2}{4}A_{18} + \frac{(r_2 r_3)^2}{4}A_{19} + \frac{[(1-r_2)r_3]^2}{4}A_{20} + \frac{1}{4}A_{37} \\
& + \frac{[(1-r_1)(1-f_2)+r_1 f_2]^2}{4}A_{38} + \frac{[(1-r_1)f_2 + r_1(1-f_2)]^2}{4}A_{39} + \frac{(1-r_1)^2}{4}A_{41} + \frac{r_1^2}{4}A_{42} \\
& + \frac{[(1-r_1)(1-f_1)]^2}{4}A_{43} + \frac{(r_1 f_1)^2}{4}A_{44} + \frac{[r_1(1-f_1)]^2}{4}A_{45} + \frac{[(1-r_1)f_1]^2}{4}A_{46} + \frac{(1-f_1)^2}{4}A_{53} \\
& + \frac{f_1^2}{4}A_{54} + \frac{[(1-f_1)(1-r_3)]^2}{4}A_{55} + \frac{[(1-r_3)f_1]^2}{4}A_{56} + \frac{[r_3(1-f_1)]^2}{4}A_{57} + \frac{(r_3 f_1)^2}{4}A_{58}
\end{aligned}
$$

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$$+ \frac{[(1-r_1)(1-r_2)]^2}{4}A_{61} + \frac{(r_1 r_2)^2}{4}A_{62} + \frac{[(1-r_1)r_2]^2}{4}A_{63} + \frac{[r_1(1-r_2)]^2}{4}A_{64}$$

$$+ \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{65} + \frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{66} + \frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{67}$$

$$+ \frac{(r_1 r_2 r_3)^2}{4}A_{68} + \frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{69} + \frac{[r_1 r_2(1-r_3)]^2}{4}A_{70} + \frac{[r_1(1-r_2)r_3]^2}{4}A_{71}$$

$$+ \frac{[(1-r_1)r_2 r_3]^2}{4}A_{72},$$

where A_1' denotes the frequency of $\frac{1111}{1111}$ genotype in the next generation, $f_1 = r_1(1-r_2) + r_2(1-r_1)$ and $f_2 = r_2(1-r_3) + r_3(1-r_2)$ $(1-f_1 = (1-r_1)(1-r_2) + r_1 r_2$ and $1-f_2 = (1-r_2)(1-r_3) + r_2 r_3)$.

The other 71 equations for the remaining genotypic frequencies are given below.

$$\begin{aligned}
A_2' =\;& \frac{1}{2}A_2 + \frac{f_2(1-f_2)}{2}A_5 + \frac{f_2(1-f_2)}{2}A_6 + \frac{r_3(1-r_3)}{2}A_{12} + \frac{r_3(1-r_3)}{2}A_{13} \\
&+ \frac{(1-r_2)^2 r_3(1-r_3)}{2}A_{17} + \frac{r_2^2 r_3(1-r_3)}{2}A_{18} + \frac{r_2^2 r_3(1-r_3)}{2}A_{19} + \frac{(1-r_2)^2 r_3(1-r_3)}{2}A_{20} \\
&+ \frac{[(1-r_1)(1-f_2) + r_1 f_2][(1-r_1)f_2 + r_1(1-f_2)]}{2}(A_{38} + A_{39}) \\
&+ \frac{(1-r_1)^2 f_1(1-f_1)}{2}A_{43} + \frac{r_1^2 f_1(1-f_1)}{2}A_{44} + \frac{r_1^2 f_1(1-f_1)}{2}A_{45} + \frac{(1-r_1)^2 f_1(1-f_1)}{2}A_{46} \\
&+ \frac{(1-f_1)^2 r_3(1-r_3)}{2}A_{55} + \frac{f_1^2 r_3(1-r_3)}{2}A_{56} + \frac{(1-f_1)^2 r_3(1-r_3)}{2}A_{57} + \frac{f_1^2 r_3(1-r_3)}{2}A_{58} \\
&+ \frac{[(1-r_1)(1-r_2)]^2 r_3(1-r_3)}{2}A_{65} + \frac{[r_1(1-r_2)]^2 r_3(1-r_3)}{2}A_{66} + \frac{[(1-r_1)r_2]^2 r_3(1-r_3)}{2}A_{67} \\
&+ \frac{(r_1 r_2)^2 r_3(1-r_3)}{2}A_{68} + \frac{[(1-r_1)(1-r_2)]^2 r_3(1-r_3)}{2}A_{69} + \frac{(r_1 r_2)^2 r_3(1-r_3)}{2}A_{70} \\
&+ \frac{[r_1(1-r_2)]^2 r_3(1-r_3)}{2}A_{71} + \frac{[(1-r_1)r_2]^2 r_3(1-r_3)}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A_3' =\;& \frac{1}{4}A_2 + A_3 + \frac{f_2^2}{4}A_5 + \frac{(1-f_2)^2}{4}A_6 + \frac{1}{4}A_7 + \frac{r_3^2}{4}A_{12} + \frac{(1-r_3)^2}{4}A_{13} + \frac{1}{4}A_{14} + \frac{[(1-r_2)r_3]^2}{4}A_{17} \\
&+ \frac{(r_2 r_3)^2}{4}A_{18} + \frac{[r_2(1-r_3)]^2}{4}A_{19} + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{20} + \frac{(1-r_2)^2}{4}A_{21} + \frac{r_2^2}{4}A_{22} \\
&+ \frac{[(1-r_1)f_2 + r_1(1-f_2)]^2}{4}A_{38} + \frac{[(1-r_1)(1-f_2) + r_1 f_2]^2}{4}A_{39} + \frac{1}{4}A_{40} + \frac{[(1-r_1)f_1]^2}{4}A_{43} \\
&+ \frac{[r_1(1-f_1)]^2}{4}A_{44} + \frac{(r_1 f_1)^2}{4}A_{45} + \frac{[(1-r_1)(1-f_1)]^2}{4}A_{46} + \frac{(1-r_1)^2}{4}A_{47} + \frac{r_1^2}{4}A_{48} \\
&+ \frac{[r_3(1-f_1)]^2}{4}A_{55} + \frac{(r_3 f_1)^2}{4}A_{56} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{57} + \frac{[(1-r_3)f_1]^2}{4}A_{58} + \frac{(1-f_1)^2}{4}A_{59} \\
&+ \frac{f_1^2}{4}A_{60} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{61} + \frac{(r_1 r_2)^2}{4}A_{62} + \frac{[(1-r_1)r_2]^2}{4}A_{63} + \frac{[r_1(1-r_2)]^2}{4}A_{64} \\
&+ \frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{65} + \frac{[r_1(1-r_2)r_3]^2}{4}A_{66} + \frac{[(1-r_1)r_2 r_3]^2}{4}A_{67} + \frac{[r_1 r_2(1-r_3)]^2}{4}A_{68}
\end{aligned}$$

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$$+\frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{69} + \frac{(r_1 r_2 r_3)^2}{4}A_{70} + \frac{r_1(1-r_2)(1-r_3)]^2}{4}A_{71}$$

$$+\frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{72}$$

$$
\begin{aligned}
A_4' &= \frac{1}{2}A_4 + \frac{f_2(1-f_2)}{2}(A_5 + A_6) + \frac{r_2(1-r_2)}{2}(A_{15} + A_{16}) + \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17} + A_{18}) \\
&\quad + r_3^2(A_{19} + A_{20})] + \frac{r_1(1-r_1)}{2}(A_{41} + A_{42}) + \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) \\
&\quad + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61} + A_{62} + A_{63} + A_{64}) \\
&\quad + \frac{r_1(1-r_1)r_2(1-r_2)}{2}[(1-r_3)^2(A_{65} + A_{66} + A_{67} + A_{70}) + r_3^2(A_{68} + A_{69} + A_{71} + A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A_5' &= \frac{(1-f_2)^2}{2}A_5 + \frac{f_2^2}{2}A_6 + \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) \\
&\quad + \frac{r_1(1-r_1)}{2}[(1-f_1)^2(A_{43} + A_{45}) + f_1^2(A_{44} + A_{46})] \\
&\quad + \frac{r_1 r_2 r_3(1-r_1)(1-r_2)(1-r_3)}{2}(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})
\end{aligned}
$$

$$
\begin{aligned}
A_6' &= \frac{f_2^2}{2}A_5 + \frac{(1-f_2)^2}{2}A_6 + \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) \\
&\quad + \frac{r_1(1-r_1)}{2}[f_1^2(A_{43} + A_{45}) + (1-f_1)^2(A_{44} + A_{46})] \\
&\quad + \frac{r_1 r_2 r_3(1-r_1)(1-r_2)(1-r_3)}{2}(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})
\end{aligned}
$$

$$
\begin{aligned}
A_7' &= \frac{f_2(1-f_2)}{2}(A_5 + A_6) + \frac{1}{2}A_7 + \frac{r_2(1-r_2)}{2}[r_3^2(A_{17} + A_{18}) + (1-r_3)^2(A_{19} + A_{20})] \\
&\quad + \frac{r_2(1-r_2)}{2}(A_{21} + A_{22}) + \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) \\
&\quad + \frac{r_1(1-r_1)}{2}(A_{47} + A_{48}) + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61} + A_{62} + A_{63} + A_{64}) \\
&\quad + \frac{r_1(1-r_1)r_2(1-r_2)}{2}[r_3^2(A_{65} + A_{66} + A_{67} + A_{70}) + (1-r_3)^2(A_{68} + A_{69} + A_{71} + A_{72})]
\end{aligned}
$$

$$
A_8' = \frac{1}{4}A_4 + \frac{f_2^2}{4}A_5 + \frac{(1-f_2)^2}{4}A_6 + A_8 + \frac{1}{4}A_9 + \frac{r_2^2}{4}A_{15} + \frac{(1-r_2)^2}{4}A_{16} + \frac{[r_2(1-r_3)]^2}{4}A_{17}
$$

$$+\frac{[(1-r_2)(1-r_3)]^2}{4}A_{18} + \frac{[(1-r_2)r_3]^2}{4}A_{19} + \frac{(r_2r_3)^2}{4}A_{20} + \frac{1}{4}A_{23} + \frac{(1-r_3)^2}{4}A_{24}$$

$$+\frac{r_3^2}{4}A_{25} + \frac{r_1^2}{4}A_{41} + \frac{(1-r_1)^2}{4}A_{42} + \frac{(r_1f_1)^2}{4}A_{43} + \frac{[(1-r_1)(1-f_1)]^2}{4}A_{44}$$

$$+\frac{[(1-r_1)f_1]^2}{4}A_{45} + \frac{[r_1(1-f_1)]^2}{4}A_{46} + \frac{1}{4}A_{49} + \frac{[(1-r_1)(1-f_1)+r_1f_1]^2}{4}A_{50}$$

$$+\frac{[(1-r_1)f_1+r_1(1-f_1)]^2}{4}A_{51} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{55} + \frac{[(1-r_3)f_1]^2}{4}A_{56}$$

$$+\frac{[r_3(1-f_1)]^2}{4}A_{57} + \frac{(r_3f_1)^2}{4}A_{58} + \frac{(1-f_1)^2}{4}A_{59} + \frac{f_1^2}{4}A_{60} + \frac{(r_1r_2)^2}{4}A_{61}$$

$$+\frac{[(1-r_1)(1-r_2)]^2}{4}A_{62} + \frac{[r_1(1-r_2)]^2}{4}A_{63} + \frac{[(1-r_1)r_2]^2}{4}A_{64} + \frac{[r_1r_2(1-r_3)]^2}{4}A_{65}$$

$$+\frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{66} + \frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{67} + \frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{68}$$

$$+\frac{(r_1r_2r_3)^2}{4}A_{69} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{70} + \frac{[(1-r_1)r_2r_3]^2}{4}A_{71} + \frac{[r_1(1-r_2)r_3]^2}{4}A_{72}$$

$$
\begin{aligned}
A_9' \;=\; & \frac{f_2(1-f_2)}{2}(A_5 + A_6) + \frac{1}{2}A_9 + \frac{r_3(1-r_3)}{2}[r_2^2(A_{17}+A_{20}) + (1-r_2)^2(A_{18}+A_{19})] \\
& +\frac{r_3(1-r_3)}{2}(A_{24}+A_{25}) + \frac{f_1(1-f_1)}{2}[r_1^2(A_{43}+A_{46}) + (1-r_1)^2(A_{44}+A_{45})] \\
& +\frac{[(1-r_1)(1-f_1)+r_1f_1][(1-r_1)f_1+r_1(1-f_1)]}{2}(A_{50}+A_{51}) \\
& +\frac{r_3(1-r_3)}{2}[(1-f_1)^2(A_{55}+A_{57}) + f_1^2(A_{56}+A_{58})] \\
& +\frac{r_3(1-r_3)}{2}[r_1^2r_2^2(A_{65}+A_{69}) + (1-r_1)^2r_2^2(A_{66}+A_{71}) + r_1^2(1-r_2)^2(A_{67}+A_{72}) \\
& +(1-r_1)^2(1-r_2)^2(A_{68}+A_{70})]
\end{aligned}
$$

$$
\begin{aligned}
A_{10}' \;=\; & \frac{(1-f_2)^2}{4}A_5 + \frac{f_2^2}{4}A_6 + \frac{1}{4}A_7 + \frac{1}{4}A_9 + A_{10} + \frac{(r_2r_3)^2}{4}A_{17} + \frac{[(1-r_2)r_3]^2}{4}A_{18} \\
& +\frac{[(1-r_2)(1-r_3)]^2}{4}A_{19} + \frac{[r_2(1-r_3)]^2}{4}A_{20} + \frac{r_2^2}{4}A_{21} + \frac{(1-r_2)^2}{4}A_{22} + \frac{r_3^2}{4}A_{24} \\
& +\frac{(1-r_3)^2}{4}A_{25} + \frac{1}{4}A_{26} + \frac{[r_1(1-f_1)]^2}{4}A_{43} + \frac{[(1-r_1)f_1]^2}{4}A_{44} + \frac{[(1-r_1)(1-f_1)]^2}{4}A_{45} \\
& +\frac{(r_1f_1)^2}{4}A_{46} + \frac{r_1^2}{4}A_{47} + \frac{(1-r_1)^2}{4}A_{48} + \frac{[(1-r_1)f_1+r_1(1-f_1)]^2}{4}A_{50} \\
& +\frac{[(1-r_1)(1-f_1)+r_1f_1]^2}{4}A_{51} + \frac{1}{4}A_{52} + \frac{(1-f_1)^2}{4}A_{53} + \frac{f_1^2}{4}A_{54} + \frac{[r_3(1-f_1)]^2}{4}A_{55} \\
& +\frac{(r_3f_1)^2}{4}A_{56} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{57} + \frac{[(1-r_3)f_1]^2}{4}A_{58} + \frac{(r_1r_2)^2}{4}A_{61} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{62}
\end{aligned}
$$

$$+\frac{[r_1(1-r_2)]^2}{4}A_{63}+\frac{[(1-r_1)r_2]^2}{4}A_{64}+\frac{(r_1r_2r_3)^2}{4}A_{65}+\frac{[(1-r_1)r_2r_3]^2}{4}A_{66}+\frac{[r_1(1-r_2)r_3]^2}{4}A_{67}$$

$$+\frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{68}+\frac{[r_1r_2(1-r_3)]^2}{4}A_{69}+\frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{70}$$

$$+\frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{71}+\frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{72}$$

$$A'_{11} = \frac{1}{2}A_{11}+\frac{r_3(1-r_3)}{2}(A_{12}+A_{13})+\frac{r_2(1-r_2)}{2}(A_{15}+A_{16})$$

$$+\frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17}+A_{18}+A_{19}+A_{20})+\frac{f_1(1-f_1)}{2}(A_{53}+A_{54})$$

$$+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58})$$

$$+\frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61}+A_{63})+r_1^2(A_{62}+A_{64})]$$

$$+\frac{r_2(1-r_2)r_3(1-r_3)}{2}[(1-r_1)^2(A_{65}+A_{67}+A_{69}+A_{72})+r_1^2(A_{66}+A_{68}+A_{70}+A_{71})]$$

$$A'_{12} = \frac{(1-r_3)^2}{2}A_{12}+\frac{r_3^2}{2}A_{13}+\frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17}+A_{18})+r_3^2(A_{19}+A_{20})]$$

$$+\frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55}+A_{56})+r_3^2(A_{57}+A_{58})]$$

$$+\frac{r_2(1-r_2)}{2}[(1-r_1)^2(1-r_3)^2(A_{65}+A_{67})+r_1^2(1-r_3)^2(A_{66}+A_{70})+r_1^2r_3^2(A_{68}+A_{71})$$

$$+(1-r_1)^2r_3^2(A_{69}+A_{72})]$$

$$A'_{13} = \frac{r_3^2}{2}A_{12}+\frac{(1-r_3)^2}{2}A_{13}+\frac{r_2(1-r_2)}{2}[r_3^2(A_{17}+A_{18})+(1-r_3)^2(A_{19}+A_{20})]$$

$$+\frac{f_1(1-f_1)}{2}[r_3^2(A_{55}+A_{56})+(1-r_3)^2(A_{57}+A_{58})]$$

$$+\frac{r_2(1-r_2)}{2}[(1-r_1)^2r_3^2(A_{65}+A_{67})+r_1^2r_3^2(A_{66}+A_{70})+r_1^2(1-r_3)^2(A_{68}+A_{71})$$

$$+(1-r_1)^2(1-r_3)^2(A_{69}+A_{72})]$$

$$A'_{14} = \frac{r_3(1-r_3)}{2}(A_{12}+A_{13})+\frac{1}{2}A_{14}+\frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17}+A_{18}+A_{19}+A_{20})$$

$$+\frac{r_2(1-r_2)}{2}(A_{21}+A_{22})+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58})$$

$$+\frac{f_1(1-f_1)}{2}(A_{59}+A_{60})+\frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61}+A_{63})+r_1^2(A_{62}+A_{64})]$$

$$+\frac{r_2(1-r_2)r_3(1-r_3)}{2}[(1-r_1)^2(A_{65}+A_{67}+A_{69}+A_{72})+r_1^2(A_{66}+A_{68}+A_{70}+A_{71})]$$

$$A'_{15} = \frac{(1-r_2)^2}{2}A_{15} + \frac{r_2^2}{2}A_{16} + \frac{r_3(1-r_3)}{2}[(1-r_2)^2(A_{17}+A_{20}) + r_2^2(A_{18}+A_{19})]$$
$$+\frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61}+A_{64}) + r_2^2(A_{62}+A_{63})]$$
$$+\frac{r_1(1-r_1)r_3(1-r_3)}{2}[(1-r_2)^2(A_{65}+A_{66}+A_{69}+A_{71}) + r_2^2(A_{67}+A_{68}+A_{70}+A_{72})]$$

$$A'_{16} = \frac{r_2^2}{2}A_{15} + \frac{(1-r_2)^2}{2}A_{16} + \frac{r_3(1-r_3)}{2}[r_2^2(A_{17}+A_{20}) + (1-r_2)^2(A_{18}+A_{19})]$$
$$+\frac{r_1(1-r_1)}{2}[r_2^2(A_{61}+A_{64}) + (1-r_2)^2(A_{62}+A_{63})]$$
$$+\frac{r_1(1-r_1)r_3(1-r_3)}{2}[r_2^2(A_{65}+A_{66}+A_{69}+A_{71}) + (1-r_2)^2(A_{67}+A_{68}+A_{70}+A_{72})]$$

$$A'_{17} = \frac{[(1-r_2)(1-r_3)]^2}{2}A_{17} + \frac{[r_2(1-r_3)]^2}{2}A_{18} + \frac{(r_2r_3)^2}{2}A_{19} + \frac{[(1-r_2)r_3]^2}{2}A_{20}$$
$$+\frac{r_1(1-r_1)}{2}[(1-r_2)^2(1-r_3)^2(A_{65}+A_{66}) + r_2^2(1-r_3)^2(A_{67}+A_{70}) + r_2^2r_3^2(A_{68}+A_{72})$$
$$+(1-r_2)^2r_3^2(A_{69}+A_{71})]$$

$$A'_{18} = \frac{[r_2(1-r_3)]^2}{2}A_{17} + \frac{[(1-r_2)(1-r_3)]^2}{2}A_{18} + \frac{[(1-r_2)r_3]^2}{2}A_{19} + \frac{(r_2r_3)^2}{2}A_{20}$$
$$+\frac{r_1(1-r_1)}{2}[r_2^2(1-r_3)^2(A_{65}+A_{66}) + (1-r_2)^2(1-r_3)^2(A_{67}+A_{70})$$
$$+(1-r_2)^2r_3^2(A_{68}+A_{72}) + r_2^2r_3^2(A_{69}+A_{71})]$$

$$A'_{19} = \frac{(r_2r_3)^2}{2}A_{17} + \frac{[(1-r_2)r_3]^2}{2}A_{18} + \frac{[(1-r_2)(1-r_3)]^2}{2}A_{19} + \frac{[r_2(1-r_3)]^2}{2}A_{20}$$
$$+\frac{r_1(1-r_1)}{2}[r_2^2r_3^2(A_{65}+A_{66}) + (1-r_2)^2r_3^2(A_{67}+A_{70}) + (1-r_2)^2(1-r_3)^2(A_{68}+A_{72})$$
$$+r_2^2(1-r_3)^2(A_{69}+A_{71})]$$

$$A'_{20} = \frac{[(1-r_2)r_3]^2}{2}A_{17} + \frac{(r_2r_3)^2}{2}A_{18} + \frac{[r_2(1-r_3)]^2}{2}A_{19} + \frac{[(1-r_2)(1-r_3)]^2}{2}A_{20}$$
$$+\frac{r_1(1-r_1)}{2}[(1-r_2)^2r_3^2(A_{65}+A_{66}) + r_2^2r_3^2(A_{67}+A_{70}) + r_2^2(1-r_3)^2(A_{68}+A_{72})$$
$$+(1-r_2)^2(1-r_3)^2(A_{69}+A_{71})]$$

$$A'_{21} = \frac{r_3(1-r_3)}{2}[(1-r_2)^2(A_{17}+A_{20})+r_2^2(A_{18}+A_{19})]+\frac{(1-r_2)^2}{2}A_{21}+\frac{r_2^2}{2}A_{22}$$
$$+\frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61}+A_{64})+r_2^2(A_{62}+A_{63})]$$
$$+\frac{r_1(1-r_1)r_3(1-r_3)}{2}[(1-r_2)^2(A_{65}+A_{66}+A_{69}+A_{71})+r_2^2(A_{67}+A_{68}+A_{70}+A_{72})]$$

$$A'_{22} = \frac{r_3(1-r_3)}{2}[r_2^2(A_{17}+A_{20})+(1-r_2)^2(A_{18}+A_{19})]+\frac{r_2^2}{2}A_{21}+\frac{(1-r_2)^2}{2}A_{22}$$
$$+\frac{r_1(1-r_1)}{2}[r_2^2(A_{61}+A_{64})+(1-r_2)^2(A_{62}+A_{63})]$$
$$+\frac{r_1(1-r_1)r_3(1-r_3)}{2}[r_2^2(A_{65}+A_{66}+A_{69}+A_{71})+(1-r_2)^2(A_{67}+A_{68}+A_{70}+A_{72})]$$

$$A'_{23} = \frac{r_2(1-r_2)}{2}(A_{15}+A_{16})+\frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17}+A_{18}+A_{19}+A_{20})+\frac{1}{2}A_{23}$$
$$+\frac{r_3(1-r_3)}{2}(A_{24}+A_{25})+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58})$$
$$+\frac{f_1(1-f_1)}{2}(A_{59}+A_{60})+\frac{r_2(1-r_2)}{2}[r_1^2(A_{61}+A_{63})+(1-r_1)^2(A_{62}+A_{64})]$$
$$+\frac{r_2(1-r_2)r_3(1-r_3)}{2}[r_1^2(A_{65}+A_{67}+A_{69}+A_{72})+(1-r_1)^2(A_{66}+A_{68}+A_{70}+A_{71})]$$

$$A'_{24} = \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17}+A_{18})+r_3^2(A_{19}+A_{20})]+\frac{(1-r_3)^2}{2}A_{24}+\frac{r_3^2}{2}A_{25}$$
$$+\frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55}+A_{56})+r_3^2(A_{57}+A_{58})]$$
$$+\frac{r_2(1-r_2)}{2}[r_1^2(1-r_3)^2(A_{65}+A_{67})+(1-r_1)^2(1-r_3)^2(A_{66}+A_{70})$$
$$+(1-r_1)^2r_3^2(A_{68}+A_{71})+r_1^2r_3^2(A_{69}+A_{72})]$$

$$A'_{25} = \frac{r_2(1-r_2)}{2}[r_3^2(A_{17}+A_{18})+(1-r_3)^2(A_{19}+A_{20})]+\frac{r_3^2}{2}A_{24}+\frac{(1-r_3)^2}{2}A_{25}$$
$$+\frac{f_1(1-f_1)}{2}[r_3^2(A_{55}+A_{56})+(1-r_3)^2(A_{57}+A_{58})]$$
$$+\frac{r_2(1-r_2)}{2}[r_1^2r_3^2(A_{65}+A_{67})+(1-r_1)^2r_3^2(A_{66}+A_{70})$$
$$+(1-r_1)^2(1-r_3)^2(A_{68}+A_{71})+r_1^2(1-r_3)^2(A_{69}+A_{72})]$$

$$A'_{26} = \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17}+A_{18}+A_{19}+A_{20}) + \frac{r_2(1-r_2)}{2}(A_{21}+A_{22})$$

$$+\frac{r_3(1-r_3)}{2}(A_{24}+A_{25}) + \frac{1}{2}A_{26} + \frac{f_1(1-f_1)}{2}(A_{53}+A_{54})$$

$$+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58})$$

$$+\frac{r_2(1-r_2)}{2}[r_1^2(A_{61}+A_{63}) + (1-r_1)^2(A_{62}+A_{64})]$$

$$+\frac{r_2(1-r_2)r_3(1-r_3)}{2}[r_1^2(A_{65}+A_{67}+A_{69}+A_{72}) + (1-r_1)^2(A_{66}+A_{68}+A_{70}+A_{71})]$$

$$A'_{27} = \frac{1}{4}A_{11} + \frac{r_3^2}{4}A_{12} + \frac{(1-r_3)^2}{4}A_{13} + \frac{r_2^2}{4}A_{15} + \frac{(1-r_2)^2}{4}A_{16} + \frac{(r_2 r_3)^2}{4}A_{17} + \frac{[(1-r_2)r_3]^2}{4}A_{18}$$

$$+\frac{[(1-r_2)(1-r_3)]^2}{4}A_{19} + \frac{[r_2(1-r_3)]^2}{4}A_{20} + A_{27} + \frac{1}{4}A_{28} + \frac{1}{4}A_{30} + \frac{(1-f_2)^2}{4}A_{31} + \frac{f_2^2}{4}A_{32}$$

$$+\frac{[(1-r_1)(1-f_2)]^2}{4}A_{43} + \frac{(r_1 f_2)^2}{4}A_{44} + \frac{[r_1(1-f_2)]^2}{4}A_{45} + \frac{[(1-r_1)f_2]^2}{4}A_{46}$$

$$+\frac{(1-r_1)^2}{4}A_{47} + \frac{r_1^2}{4}A_{48} + \frac{[(1-r_1)(1-f_2)+r_1 f_2]^2}{4}A_{50} + \frac{[(1-r_1)f_2+r_1(1-f_2)]^2}{4}A_{51}$$

$$+\frac{1}{4}A_{52} + \frac{f_1^2}{4}A_{53} + \frac{(1-f_1)^2}{4}A_{54} + \frac{(r_3 f_1)^2}{4}A_{55} + \frac{[r_3(1-f_1)]^2}{4}A_{56} + \frac{[(1-r_3)f_1]^2}{4}A_{57}$$

$$+\frac{[(1-r_3)(1-f_1)]^2}{4}A_{58} + \frac{[(1-r_1)r_2]^2}{4}A_{61} + \frac{[r_1(1-r_2)]^2}{4}A_{62} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{63}$$

$$+\frac{(r_1 r_2)^2}{4}A_{64} + \frac{[(1-r_1)r_2 r_3]^2}{4}A_{65} + \frac{(r_1 r_2 r_3)^2}{4}A_{66} + \frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{67}$$

$$+\frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{68} + \frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{69} + \frac{[r_1(1-r_2)r_3]^2}{4}A_{70}$$

$$+\frac{[r_1 r_2(1-r_3)]^2}{4}A_{71} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{72}$$

$$A'_{28} = \frac{r_3(1-r_3)}{2}(A_{12}+A_{13}) + \frac{r_3(1-r_3)}{2}[r_2^2(A_{17}+A_{20}) + (1-r_2)^2(A_{18}+A_{19})] + \frac{1}{2}A_{28}$$

$$+\frac{f_2(1-f_2)}{2}(A_{31}+A_{32}) + \frac{f_2(1-f_2)}{2}[(1-r_1)^2(A_{43}+A_{46}) + r_1^2(A_{44}+A_{45})]$$

$$+\frac{[(1-r_1)(1-f_2)+r_1 f_2][r_1(1-f_2)+(1-r_1)f_2]}{2}(A_{50}+A_{51})$$

$$+\frac{r_3(1-r_3)}{2}[f_1^2(A_{55}+A_{57}) + (1-f_1)^2(A_{56}+A_{58})] + \frac{r_3(1-r_3)}{2}[(1-r_1)^2 r_2^2(A_{65}+A_{69})$$

$$+r_1^2 r_2^2(A_{66}+A_{71}) + (1-r_1)^2(1-r_2)^2(A_{67}+A_{72}) + r_1^2(1-r_2)^2(A_{68}+A_{70})]$$

$$A'_{29} = \frac{(1-r_3)^2}{4}A_{12} + \frac{r_3^2}{4}A_{13} + \frac{1}{4}A_{14} + \frac{[r_2(1-r_3)]^2}{4}A_{17} + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{18} + \frac{[(1-r_2)r_3]^2}{4}A_{19}$$
$$+ \frac{(r_2 r_3)^2}{4}A_{20} + \frac{r_2^2}{4}A_{21} + \frac{(1-r_2)^2}{4}A_{22} + \frac{1}{4}A_{28} + A_{29} + \frac{f_2^2}{4}A_{31} + \frac{(1-f_2)^2}{4}A_{32} + \frac{1}{4}A_{33}$$
$$+ \frac{(1-r_1)^2}{4}A_{41} + \frac{r_1^2}{4}A_{42} + \frac{[(1-r_1)f_2]^2}{4}A_{43} + \frac{[r_1(1-f_2)]^2}{4}A_{44} + \frac{(r_1 f_2)^2}{4}A_{45}$$
$$+ \frac{[(1-r_1)(1-f_2)]^2}{4}A_{46} + \frac{1}{4}A_{49} + \frac{[r_1(1-f_2)+(1-r_1)f_2]^2}{4}A_{50} + \frac{[(1-r_1)(1-f_2)+r_1 f_2]^2}{4}A_{51}$$
$$+ \frac{[(1-r_3)f_1]^2}{4}A_{55} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{56} + \frac{(r_3 f_1)^2}{4}A_{57} + \frac{[r_3(1-f_1)]^2}{4}A_{58} + \frac{f_1^2}{4}A_{59}$$
$$+ \frac{(1-f_1)^2}{4}A_{60} + \frac{[(1-r_1)r_2]^2}{4}A_{61} + \frac{[r_1(1-r_2)]^2}{4}A_{62} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{63} + \frac{(r_1 r_2)^2}{4}A_{64}$$
$$+ \frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{65} + \frac{[r_1 r_2(1-r_3)]^2}{4}A_{66} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{67}$$
$$+ \frac{[r_1(1-r_2)r_3]^2}{4}A_{68} + \frac{[(1-r_1)r_2 r_3]^2}{4}A_{69} + \frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{70} + \frac{(r_1 r_2 r_3)^2}{4}A_{71}$$
$$+ \frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{72}$$

$$A'_{30} = \frac{r_2(1-r_2)}{2}(A_{15}+A_{16}) + \frac{r_2(1-r_2)}{2}[r_3^2(A_{17}+A_{18}) + (1-r_3)^2(A_{19}+A_{20})] + \frac{1}{2}A_{30}$$
$$+ \frac{f_2(1-f_2)}{2}(A_{31}+A_{32}) + \frac{r_1(1-r_1)f_2(1-f_2)}{2}(A_{43}+A_{44}+A_{45}+A_{46})$$
$$+ \frac{r_1(1-r_1)}{2}(A_{47}+A_{48}) + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64})$$
$$+ \frac{r_1(1-r_1)r_2(1-r_2)}{2}[r_3^2(A_{65}+A_{66}+A_{67}+A_{70}) + (1-r_3)^2(A_{68}+A_{69}+A_{71}+A_{72})]$$

$$A'_{31} = \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17}+A_{18}+A_{19}+A_{20}) + \frac{(1-f_2)^2}{2}A_{31} + \frac{f_2^2}{2}A_{32}$$
$$+ \frac{r_1(1-r_1)}{2}[(1-f_2)^2(A_{43}+A_{45}) + f_2^2(A_{44}+A_{46})]$$
$$+ \frac{r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)}{2}(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})$$

$$A'_{32} = \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17}+A_{18}+A_{19}+A_{20}) + \frac{f_2^2}{2}A_{31} + \frac{(1-f_2)^2}{2}A_{32}$$
$$+ \frac{r_1(1-r_1)}{2}[f_2^2(A_{43}+A_{45}) + (1-f_2)^2(A_{44}+A_{46})]$$
$$+ \frac{r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)}{2}(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})$$

$$A'_{33} = \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17}+A_{18})+r_3^2(A_{19}+A_{20})]+\frac{r_2(1-r_2)}{2}(A_{21}+A_{22})$$

$$+\frac{f_2(1-f_2)}{2}(A_{31}+A_{32})+\frac{1}{2}A_{33}+\frac{r_1(1-r_1)}{2}(A_{41}+A_{42})$$

$$+\frac{r_1(1-r_1)f_2(1-f_2)}{2}(A_{43}+A_{44}+A_{45}+A_{46})+\frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64})$$

$$+\frac{r_1(1-r_1)r_2(1-r_2)}{2}[(1-r_3)^2(A_{65}+A_{66}+A_{67}+A_{70})+r_3^2(A_{68}+A_{69}+A_{71}+A_{72})]$$

$$A'_{34} = \frac{(1-r_2)^2}{4}A_{15}+\frac{r_2^2}{4}A_{16}+\frac{[(1-r_2)r_3]^2}{4}A_{17}+\frac{(r_2r_3)^2}{4}A_{18}+\frac{[r_2(1-r_3)]^2}{4}A_{19}$$

$$+\frac{[(1-r_2)(1-r_3)]^2}{4}A_{20}+\frac{1}{4}A_{23}+\frac{r_3^2}{4}A_{24}+\frac{(1-r_3)^2}{4}A_{25}+\frac{1}{4}A_{30}+\frac{f_2^2}{4}A_{31}+\frac{(1-f_2)^2}{4}A_{32}$$

$$+A_{34}+\frac{1}{4}A_{35}+\frac{[(1-r_1)(1-f_2)+r_1f_2]^2}{4}A_{38}+\frac{[r_1(1-f_2)+(1-r_1)f_2]^2}{4}A_{39}+\frac{1}{4}A_{40}$$

$$+\frac{(r_1f_2)^2}{4}A_{43}+\frac{[(1-r_1)(1-f_2)]^2}{4}A_{44}+\frac{[(1-r_1)f_2]^2}{4}A_{45}+\frac{[r_1(1-f_2)]^2}{4}A_{46}$$

$$+\frac{r_1^2}{4}A_{47}+\frac{(1-r_1)^2}{4}A_{48}+\frac{(r_3f_1)^2}{4}A_{55}+\frac{[r_3(1-f_1)]^2}{4}A_{56}+\frac{[(1-r_3)f_1]^2}{4}A_{57}$$

$$+\frac{[(1-r_3)(1-f_1)]^2}{4}A_{58}+\frac{f_1^2}{4}A_{59}+\frac{(1-f_1)^2}{4}A_{60}+\frac{[r_1(1-r_2)]^2}{4}A_{61}+\frac{[(1-r_1)r_2]^2}{4}A_{62}$$

$$+\frac{(r_1r_2)^2}{4}A_{63}+\frac{[(1-r_1)(1-r_2)]^2}{4}A_{64}+\frac{[r_1(1-r_2)r_3]^2}{4}A_{65}+\frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{66}$$

$$+\frac{(r_1r_2r_3)^2}{4}A_{67}+\frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{68}+\frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{69}+\frac{[(1-r_1)r_2r_3]^2}{4}A_{70}$$

$$+\frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{71}+\frac{[r_1r_2(1-r_3)]^2}{4}A_{72}$$

$$A'_{35} = \frac{r_3(1-r_3)}{2}[(1-r_2)^2(A_{17}+A_{20})+r_2^2(A_{18}+A_{19})]+\frac{r_3(1-r_3)}{2}(A_{24}+A_{25})$$

$$+\frac{f_2(1-f_2)}{2}(A_{31}+A_{32})+\frac{1}{2}A_{35}+\frac{[(1-r_1)(1-f_2)+r_1f_2][r_1(1-f_2)+(1-r_1)f_2]}{2}(A_{38}+A_{39})$$

$$+\frac{f_2(1-f_2)}{2}[r_1^2(A_{43}+A_{46})+(1-r_1)^2(A_{44}+A_{45})]$$

$$+\frac{r_3(1-r_3)}{2}[f_1^2(A_{55}+A_{57})+(1-f_1)^2(A_{56}+A_{58})]$$

$$+\frac{r_3(1-r_3)}{2}[r_1^2(1-r_2)^2(A_{65}+A_{69})+(1-r_1)^2(1-r_2)^2(A_{66}+A_{71})+r_1^2r_2^2(A_{67}+A_{72})$$

$$+(1-r_1)^2r_2^2(A_{68}+A_{70})]$$

$$A'_{36} = \frac{[(1-r_2)(1-r_3)]^2}{4}A_{17} + \frac{[r_2(1-r_3)]^2}{4}A_{18} + \frac{(r_2 r_3)^2}{4}A_{19} + \frac{[(1-r_2)r_3]^2}{4}A_{20}$$
$$+ \frac{(1-r_2)^2}{4}A_{21} + \frac{r_2^2}{4}A_{22} + \frac{(1-r_3)^2}{4}A_{24} + \frac{r_3^2}{4}A_{25} + \frac{1}{4}A_{26} + \frac{(1-f_2)^2}{4}A_{31} + \frac{f_2^2}{4}A_{32} + \frac{1}{4}A_{33}$$
$$+ \frac{1}{4}A_{35} + A_{36} + \frac{1}{4}A_{37} + \frac{[r_1(1-f_2)+(1-r_1)f_2]^2}{4}A_{38} + \frac{[(1-r_1)(1-f_2)+r_1 f_2]^2}{4}A_{39}$$
$$+ \frac{r_1^2}{4}A_{41} + \frac{(1-r_1)^2}{4}A_{42} + \frac{[r_1(1-f_2)]^2}{4}A_{43} + \frac{[(1-r_1)f_2]^2}{4}A_{44} + \frac{[(1-r_1)(1-f_2)]^2}{4}A_{45}$$
$$+ \frac{(r_1 f_2)^2}{4}A_{46} + \frac{f_1^2}{4}A_{53} + \frac{(1-f_1)^2}{4}A_{54} + \frac{[(1-r_3)f_1]^2}{4}A_{55} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{56} + \frac{(r_3 f_1)^2}{4}A_{57}$$
$$+ \frac{[r_3(1-f_1)]^2}{4}A_{58} + \frac{[r_1(1-r_2)]^2}{4}A_{61} + \frac{[(1-r_1)r_2]^2}{4}A_{62} + \frac{(r_1 r_2)^2}{4}A_{63} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{64}$$
$$+ \frac{[r_1(1-r_2)(1-r_3)]^2}{4}A_{65} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{4}A_{66} + \frac{[r_1 r_2(1-r_3)]^2}{4}A_{67}$$
$$+ \frac{[(1-r_1)r_2 r_3]^2}{4}A_{68} + \frac{[r_1(1-r_2)r_3]^2}{4}A_{69} + \frac{[(1-r_1)r_2(1-r_3)]^2}{4}A_{70} + \frac{[(1-r_1)(1-r_2)r_3]^2}{4}A_{71}$$
$$+ \frac{(r_1 r_2 r_3)^2}{4}A_{72}$$

$$A'_{37} = \frac{1}{2}A_{37} + \frac{[(1-r_1)(1-f_2)+r_1 f_2][r_1(1-f_2)+(1-r_1)f_2]}{2}(A_{38}+A_{39}) + \frac{r_1(1-r_1)}{2}(A_{41}+A_{42})$$
$$+ \frac{r_1(1-r_1)}{2}[(1-f_1)^2(A_{43}+A_{45}) + f_1^2(A_{44}+A_{46})] + \frac{f_1(1-f_1)}{2}(A_{53}+A_{54})$$
$$+ \frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55}+A_{56}) + r_3^2(A_{57}+A_{58})]$$
$$+ \frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61}+A_{64}) + r_2^2(A_{62}+A_{63})]$$
$$+ \frac{r_1(1-r_1)}{2}[(1-r_2)^2(1-r_3)^2(A_{65}+A_{66}) + r_2^2(1-r_3)^2(A_{67}+A_{70}) + r_2^2 r_3^2(A_{68}+A_{72})$$
$$+ (1-r_2)^2 r_3^2(A_{69}+A_{71})]$$

$$A'_{38} = \frac{[(1-r_1)(1-f_2)+r_1 f_2]^2}{2}A_{38} + \frac{[r_1(1-f_2)+(1-r_1)f_2]^2}{2}A_{39}$$
$$+ \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43}+A_{44}+A_{45}+A_{46}) + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58})$$
$$+ \frac{r_1(1-r_1)r_3(1-r_3)}{2}[(1-r_2)^2(A_{65}+A_{66}+A_{69}+A_{71}) + r_2^2(A_{67}+A_{68}+A_{70}+A_{72})]$$

$$A'_{39} = \frac{[r_1(1-f_2)+(1-r_1)f_2]^2}{2}A_{38} + \frac{[(1-r_1)(1-f_2)+r_1 f_2]^2}{2}A_{39}$$

$$+\frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43}+A_{44}+A_{45}+A_{46})+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58})$$

$$+\frac{r_1(1-r_1)r_3(1-r_3)}{2}[(1-r_2)^2(A_{65}+A_{66}+A_{69}+A_{71})+r_2^2(A_{67}+A_{68}+A_{70}+A_{72})]$$

$$\begin{aligned}
A'_{40} =\ & \frac{[(1-r_1)(1-f_2)+r_1f_2][r_1(1-f_2)+(1-r_1)f_2]}{2}(A_{38}+A_{39})+\frac{1}{2}A_{40}\\
& +\frac{r_1(1-r_1)}{2}[f_1^2(A_{43}+A_{45})+(1-f_1)^2(A_{44}+A_{46})]+\frac{r_1(1-r_1)}{2}(A_{47}+A_{48})\\
& +\frac{f_1(1-f_1)}{2}[r_3^2(A_{55}+A_{56})+(1-r_3)^2(A_{57}+A_{58})]+\frac{f_1(1-f_1)}{2}(A_{59}+A_{60})\\
& +\frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61}+A_{64})+r_2^2(A_{62}+A_{63})]\\
& +\frac{r_1(1-r_1)}{2}[(1-r_2)^2r_3^2(A_{65}+A_{66})+r_2^2r_3^2(A_{67}+A_{70})+r_2^2(1-r_3)^2(A_{68}+A_{72})\\
& +(1-r_2)^2(1-r_3)^2(A_{69}+A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{41} =\ & \frac{(1-r_1)^2}{2}A_{41}+\frac{r_1^2}{2}A_{42}+\frac{f_1(1-f_1)}{2}[(1-r_1)^2(A_{43}+A_{46})+r_1^2(A_{44}+A_{45})]\\
& +\frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61}+A_{63})+r_1^2(A_{62}+A_{64})]\\
& +\frac{r_2(1-r_2)}{2}[(1-r_1)^2(1-r_3)^2(A_{65}+A_{67})+r_1^2(1-r_3)^2(A_{66}+A_{70})\\
& +r_1^2r_3^2(A_{68}+A_{71})+(1-r_1)^2r_3^2(A_{69}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{42} =\ & \frac{r_1^2}{2}A_{41}+\frac{(1-r_1)^2}{2}A_{42}+\frac{f_1(1-f_1)}{2}[r_1^2(A_{43}+A_{46})+(1-r_1)^2(A_{44}+A_{45})]\\
& +\frac{r_2(1-r_2)}{2}[r_1^2(A_{61}+A_{63})+(1-r_1)^2(A_{62}+A_{64})]\\
& +\frac{r_2(1-r_2)}{2}[r_1^2(1-r_3)^2(A_{65}+A_{67})+(1-r_1)^2(1-r_3)^2(A_{66}+A_{70})\\
& +(1-r_1)^2r_3^2(A_{68}+A_{71})+r_1^2r_3^2(A_{69}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{43} =\ & \frac{[(1-r_1)(1-f_1)]^2}{2}A_{43}+\frac{(r_1f_1)^2}{2}A_{44}+\frac{[r_1(1-r_1)]^2}{2}A_{45}+\frac{[(1-r_1)f_1]^2}{2}A_{46}\\
& +\frac{r_2(1-r_2)r_3(1-r_3)}{2}[(1-r_1)^2(A_{65}+A_{67}+A_{69}+A_{72})+r_1^2(A_{66}+A_{68}+A_{70}+A_{71})]
\end{aligned}$$

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$$A'_{44} = \frac{(r_1 f_1)^2}{2} A_{43} + \frac{[(1-r_1)(1-f_1)]^2}{2} A_{44} + \frac{[(1-r_1)f_1]^2}{2} A_{45} + \frac{[r_1(1-f_1)]^2}{2} A_{46}$$
$$+ \frac{r_2(1-r_2)r_3(1-r_3)}{2}[r_1^2(A_{65} + A_{67} + A_{69} + A_{72}) + (1-r_1)^2(A_{66} + A_{68} + A_{70} + A_{71})]$$

$$A'_{45} = \frac{[r_1(1-f_1)]^2}{2} A_{43} + \frac{[(1-r_1)f_1]^2}{2} A_{44} + \frac{[(1-r_1)(1-f_1)]^2}{2} A_{45} + \frac{(r_1 f_1)^2}{2} A_{46}$$
$$+ \frac{r_2(1-r_2)r_3(1-r_3)}{2}[r_1^2(A_{65} + A_{67} + A_{69} + A_{72}) + (1-r_1)^2(A_{66} + A_{68} + A_{70} + A_{71})]$$

$$A'_{46} = \frac{[(1-r_1)f_1]^2}{2} A_{43} + \frac{[r_1(1-f_1)]^2}{2} A_{44} + \frac{(r_1 f_1)^2}{2} A_{45} + \frac{[(1-r_1)(1-f_1)]^2}{2} A_{46}$$
$$+ \frac{r_2(1-r_2)r_3(1-r_3)}{2}[(1-r_1)^2(A_{65} + A_{67} + A_{69} + A_{72}) + r_1^2(A_{66} + A_{68} + A_{70} + A_{71})]$$

$$A'_{47} = \frac{f_1(1-f_1)}{2}[(1-r_1)^2(A_{43} + A_{46}) + r_1^2(A_{44} + A_{45})] + \frac{(1-r_1)^2}{2} A_{47} + \frac{r_1^2}{2} A_{48}$$
$$+ \frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61} + A_{63}) + r_1^2(A_{62} + A_{64})]$$
$$+ \frac{r_2(1-r_2)}{2}[(1-r_1)^2 r_3^2(A_{65} + A_{67}) + r_1^2 r_3^2(A_{66} + A_{70}) + r_1^2(1-r_3)^2(A_{68} + A_{71})$$
$$+ (1-r_1)^2(1-r_3)^2(A_{69} + A_{72})]$$

$$A'_{48} = \frac{f_1(1-f_1)}{2}[r_1^2(A_{43} + A_{46}) + (1-r_1)^2(A_{44} + A_{45})] + \frac{r_1^2}{2} A_{47} + \frac{(1-r_1)^2}{2} A_{48}$$
$$+ \frac{r_2(1-r_2)}{2}[r_1^2(A_{61} + A_{63}) + (1-r_1)^2(A_{62} + A_{64})]$$
$$+ \frac{r_2(1-r_2)}{2}[r_1^2 r_3^2(A_{65} + A_{67}) + (1-r_1)^2 r_3^2(A_{66} + A_{70}) + (1-r_1)^2(1-r_3)^2(A_{68} + A_{71})$$
$$+ r_1^2(1-r_3)^2(A_{69} + A_{72})]$$

$$A'_{49} = \frac{r_1(1-r_1)}{2}(A_{41} + A_{42}) + \frac{r_1(1-r_1)}{2}[f_1^2(A_{43} + A_{45}) + (1-f_1)^2(A_{44} + A_{46})]$$
$$+ \frac{1}{2} A_{49} + \frac{[(1-r_1)(1-f_1) + r_1 f_1][(1-r_1)f_1 + r_1(1-f_1)]}{2}(A_{50} + A_{51})$$

$$+\frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55}+A_{56})+r_3^2(A_{57}+A_{58})]+\frac{f_1(1-f_1)}{2}(A_{59}+A_{60})$$

$$+\frac{r_1(1-r_1)}{2}[r_2^2(A_{61}+A_{64})+(1-r_2)^2(A_{62}+A_{63})]$$

$$+\frac{r_1(1-r_1)}{2}[r_2^2(1-r_3)^2(A_{65}+A_{66})+(1-r_2)^2(1-r_3)^2(A_{67}+A_{70})$$

$$+(1-r_2)^2r_3^2(A_{68}+A_{72})+r_2^2r_3^2(A_{69}+A_{71})]$$

$$
\begin{aligned}
A'_{50} =\ & \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43}+A_{44}+A_{45}+A_{46})+\frac{[(1-r_1)(1-f_1)+r_1f_1]^2}{2}A_{50} \\
& +\frac{[(1-r_1)f_1+r_1(1-f_1)]^2}{2}A_{51}+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58}) \\
& +\frac{r_1(1-r_1)r_3(1-r_3)}{2}[r_2^2(A_{65}+A_{66}+A_{69}+A_{71})+(1-r_2)^2(A_{67}+A_{68}+A_{70}+A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{51} =\ & \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43}+A_{44}+A_{45}+A_{46})+\frac{[(1-r_1)f_1+r_1(1-f_1)]^2}{2}A_{50} \\
& +\frac{[(1-r_1)(1-f_1)+r_1f_1]^2}{2}A_{51}+\frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58}) \\
& +\frac{r_1(1-r_1)r_3(1-r_3)}{2}[r_2^2(A_{65}+A_{66}+A_{69}+A_{71})+(1-r_2)^2(A_{67}+A_{68}+A_{70}+A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{52} =\ & \frac{r_1(1-r_1)}{2}[(1-f_1)^2(A_{43}+A_{45})+f_1^2(A_{44}+A_{46})]+\frac{r_1(1-r_1)}{2}(A_{47}+A_{48}) \\
& +\frac{[(1-r_1)(1-f_1)+r_1f_1][(1-r_1)f_1+r_1(1-f_1)]}{2}(A_{50}+A_{51})+\frac{1}{2}A_{52} \\
& +\frac{f_1(1-f_1)}{2}(A_{53}+A_{54})+\frac{f_1(1-f_1)}{2}[r_3^2(A_{55}+A_{56})+(1-r_3)^2(A_{57}+A_{58})] \\
& +\frac{r_1(1-r_1)}{2}[r_2^2(A_{61}+A_{64})+(1-r_2)^2(A_{62}+A_{63})]+\frac{r_1(1-r_1)}{2}[r_2^2r_3^2(A_{65}+A_{66}) \\
& +(1-r_2)^2r_3^2(A_{67}+A_{70})+(1-r_2)^2(1-r_3^2)(A_{68}+A_{72})+r_2^2(1-r_3)^2(A_{69}+A_{71})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{53} =\ & \frac{(1-f_1)^2}{2}A_{53}+\frac{f_1^2}{2}A_{54}+\frac{r_3(1-r_3)}{2}[(1-f_1)^2(A_{55}+A_{57})+f_1^2(A_{56}+A_{58})] \\
& +\frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64}) \\
& +\frac{r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)}{2}(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})
\end{aligned}
$$

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$$
\begin{aligned}
A'_{54} \;=\;& \frac{f_1^2}{2}A_{53} + \frac{(1-f_1)^2}{2}A_{54} + \frac{r_3(1-r_3)}{2}[f_1^2(A_{55}+A_{57}) + (1-f_1)^2(A_{56}+A_{58})] \\
& + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64}) \\
& + \frac{r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)}{2}(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})
\end{aligned}
$$

$$
\begin{aligned}
A'_{55} \;=\;& \frac{[(1-r_3)(1-f_1)]^2}{2}A_{55} + \frac{[(1-r_3)f_1]^2}{2}A_{56} + \frac{[r_3(1-f_1)]^2}{2}A_{57} + \frac{(r_3 f_1)^2}{2}A_{58} \\
& + \frac{r_1(1-r_1)r_2(1-r_2)}{2}[(1-r_3)^2(A_{65}+A_{66}+A_{67}+A_{70}) + r_3^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{56} \;=\;& \frac{[(1-r_3)f_1]^2}{2}A_{55} + \frac{[(1-r_3)(1-f_1)]^2}{2}A_{56} + \frac{(r_3 f_1)^2}{2}A_{57} + \frac{[r_3(1-f_1)]^2}{2}A_{58} \\
& + \frac{r_1(1-r_1)r_2(1-r_2)}{2}[(1-r_3)^2(A_{65}+A_{66}+A_{67}+A_{70}) + r_3^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{57} \;=\;& \frac{[r_3(1-f_1)]^2}{2}A_{55} + \frac{(r_3 f_1)^2}{2}A_{56} + \frac{[(1-r_3)(1-f_1)]^2}{2}A_{57} + \frac{[(1-r_3)f_1]^2}{2}A_{58} \\
& + \frac{r_1(1-r_1)r_2(1-r_2)}{2}[r_3^2(A_{65}+A_{66}+A_{67}+A_{70}) + (1-r_3)^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{58} \;=\;& \frac{(r_3 f_1)^2}{2}A_{55} + \frac{[r_3(1-f_1)]^2}{2}A_{56} + \frac{[(1-r_3)f_1]^2}{2}A_{57} + \frac{[(1-r_3)(1-f_1)]^2}{2}A_{58} \\
& + \frac{r_1(1-r_1)r_2(1-r_2)}{2}[r_3^2(A_{65}+A_{66}+A_{67}+A_{70}) + (1-r_3)^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}
$$

$$
\begin{aligned}
A'_{59} \;=\;& \frac{r_3(1-r_3)}{2}[(1-f_1)^2(A_{55}+A_{57}) + f_1^2(A_{56}+A_{58})] + \frac{(1-f_1)^2}{2}A_{59} + \frac{f_1^2}{2}A_{60} \\
& + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64}) \\
& + \frac{r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)}{2}(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})
\end{aligned}
$$

$$A'_{60} = \frac{r_3(1-r_3)}{2}[f_1^2(A_{55}+A_{57})+(1-f_1)^2(A_{56}+A_{58})]+\frac{f_1^2}{2}A_{59}+\frac{(1-f_1)^2}{2}A_{60}$$
$$+\frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64})$$
$$+\frac{r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)}{2}(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})$$

$$A'_{61} = \frac{[(1-r_1)(1-r_2)]^2}{2}A_{61}+\frac{(r_1r_2)^2}{2}A_{62}+\frac{[(1-r_1)r_2]^2}{2}A_{63}+\frac{[r_1(1-r_2)]^2}{2}A_{64}$$
$$+\frac{r_3(1-r_3)}{2}[(1-r_1)^2(1-r_2)^2(A_{65}+A_{69})+r_1^2(1-r_2)^2(A_{66}+A_{71})+r_1^2r_2^2(A_{68}+A_{70})$$
$$+(1-r_1)^2r_2^2(A_{67}+A_{72})]$$

$$A'_{62} = \frac{(r_1r_2)^2}{2}A_{61}+\frac{[(1-r_1)(1-r_2)]^2}{2}A_{62}+\frac{[r_1(1-r_2)]^2}{2}A_{63}+\frac{[(1-r_1)r_2]^2}{2}A_{64}$$
$$+\frac{r_3(1-r_3)}{2}[r_1^2r_2^2(A_{65}+A_{69})+(1-r_1)^2r_2^2(A_{66}+A_{71})+r_1^2(1-r_2)^2(A_{67}+A_{72})$$
$$+(1-r_1)^2(1-r_2)^2(A_{68}+A_{70})]$$

$$A'_{63} = \frac{[(1-r_1)r_2]^2}{2}A_{61}+\frac{[r_1(1-r_2)]^2}{2}A_{62}+\frac{[(1-r_1)(1-r_2)]^2}{2}A_{63}+\frac{(r_1r_2)^2}{2}A_{64}$$
$$+\frac{r_3(1-r_3)}{2}[(1-r_1)^2r_2^2(A_{65}+A_{69})+r_1^2r_2^2(A_{66}+A_{71})+(1-r_1)^2(1-r_2)^2(A_{67}+A_{72})$$
$$+r_1^2(1-r_2)^2(A_{68}+A_{70})]$$

$$A'_{64} = \frac{[r_1(1-r_2)]^2}{2}A_{61}+\frac{[(1-r_1)r_2]^2}{2}A_{62}+\frac{(r_1r_2)^2}{2}A_{63}+\frac{[(1-r_1)(1-r_2)]^2}{2}A_{64}$$
$$+\frac{r_3(1-r_3)}{2}[r_1^2(1-r_2)^2(A_{65}+A_{69})+(1-r_1)^2(1-r_2)^2(A_{66}+A_{71})+r_1^2r_2^2(A_{67}+A_{72})$$
$$+(1-r_1)^2r_2^2(A_{68}+A_{70})]$$

$$A'_{65} = \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{65}+\frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{66}+\frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{67}$$
$$+\frac{(r_1r_2r_3)^2}{2}A_{68}+\frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{69}+\frac{[r_1r_2(1-r_3)]^2}{2}A_{70}+\frac{[r_1(1-r_2)r_3]^2}{2}A_{71}$$
$$+\frac{[(1-r_1)r_2r_3]^2}{2}A_{72}$$

$$A'_{66} = \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{65} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{66} + \frac{[r_1 r_2(1-r_3)]^2}{2}A_{67}$$
$$+ \frac{[(1-r_1)r_2 r_3]^2}{2}A_{68} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{69} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{70}$$
$$+ \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{71} + \frac{(r_1 r_2 r_3)^2}{2}A_{72}$$

$$A'_{67} = \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{65} + \frac{[r_1 r_2(1-r_3)]^2}{2}A_{66} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{67}$$
$$+ \frac{[(r_1(1-r_2)r_3]^2}{2}A_{68} + \frac{[(1-r_1)r_2 r_3]^2}{2}A_{69} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{70} + \frac{(r_1 r_2 r_3)^2}{2}A_{71}$$
$$+ \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{72}$$

$$A'_{68} = \frac{(r_1 r_2 r_3)^2}{2}A_{65} + \frac{[(1-r_1)r_2 r_3]^2}{2}A_{66} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{67} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{68}$$
$$+ \frac{[r_1 r_2(1-r_3)]^2}{2}A_{69} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{70} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{71}$$
$$+ \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{72}$$

$$A'_{69} = \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{65} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{66} + \frac{[(1-r_1)r_2 r_3]^2}{2}A_{67} + \frac{[r_1 r_2(1-r_3)]^2}{2}A_{68}$$
$$+ \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{69} + \frac{(r_1 r_2 r_3)^2}{2}A_{70} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{71}$$
$$+ \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{72}$$

$$A'_{70} = \frac{[r_1 r_2(1-r_3)]^2}{2}A_{65} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{66} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{67}$$
$$+ \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{68} + \frac{(r_1 r_2 r_3)^2}{2}A_{69} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{70}$$
$$+ \frac{[(1-r_1)r_2 r_3]^2}{2}A_{71} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{72}$$

$$A'_{71} = \frac{[r_1(1-r_2)r_3]^2}{2}A_{65} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{66} + \frac{(r_1 r_2 r_3)^2}{2}A_{67} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{68}$$

$$+ \frac{[r_1(1-r_2)(1-r_3)]^2}{2} A_{69} + \frac{[(1-r_1)r_2 r_3]^2}{2} A_{70} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{71}$$

$$+ \frac{[r_1 r_2(1-r_3)]^2}{2} A_{72}$$

$$
\begin{aligned}
A'_{72} \quad = \quad & \frac{[(1-r_1)r_2 r_3]^2}{2} A_{65} + \frac{(r_1 r_2 r_3)^2}{2} A_{66} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2} A_{67} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2} A_{68} \\
& + \frac{[(1-r_1)r_2(1-r_3)]^2}{2} A_{69} + \frac{[r_1(1-r_2)r_3]^2}{2} A_{70} + \frac{[r_1 r_2(1-r_3)]^2}{2} A_{71} \\
& + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{72}
\end{aligned}
$$